How to Place Leakage and Wiring Inductances in the High Frequency Circuit Model

by Lloyd Dixon

TOPIC 7

How to Put Leakage and Wiring Inductances in the High Frequency Circuit Model

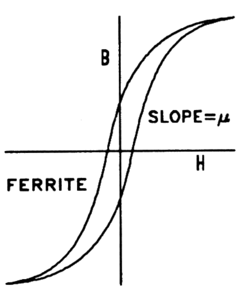
Lloyd H. Dixon, Jr.

OBJECTIVES

1. DEFINE THE ELECTRICAL CIRCUIT EQUIVALENTS OF MAGNETIC DEVICE STRUCTURES TO ENABLE IMPROVED ANALYSIS OF CIRCUIT PERFORMANCE.
2. DEFINE THE MAGNITUDE AND CIRCUIT LOCATION OF RELEVANT PARASITIC MAGNETIC ELEMENTS TO ENABLE PREDICTION OF PERFORMANCE EFFECTS.
3. MANIPULATE PARASITIC ELEMENTS TO OBTAIN IMPROVED OR ENHANCED CIRCUIT PERFORMANCE.
4. ENCOURAGE THE CIRCUIT DESIGNER TO BE MORE INVOLVED WITH MAGNETIC DEVICE DESIGN.

1A

MAGNETIC FUNDAMENTALS



INT'L STD UNITS (S.I.)
RATIONALIZED MKS

UNIT VOLUME PARAMETERS:
- B Flux Density
- H Field Intens. A-T/m
- μ Permeability B/H

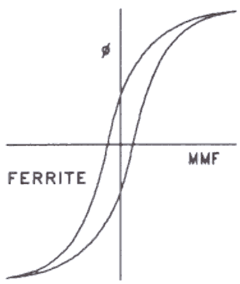
ENERGY DENSITY:
$W/m^3 = \int HdB \;\text{-->}\; BH/2$

CAN'T FORCE INST. CHANGE

AIR vs. FERRITE

1B

MAGNETIC FUNDAMENTALS



ø=BA	Flux	Webers
MMF=Hℓ	Potential	A–T

Slope:

P=ø/MMF Permeance

AMPERE'S LAW:

NI = ∫Hdℓ ––> Hℓ

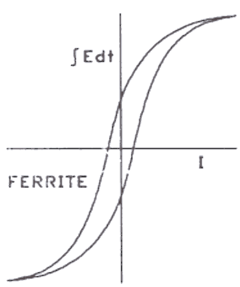
FARADAY'S LAW:

dø/dt=–E/N ; ø=1/N∫Edt

PERMEANCE IS INDUCTANCE OF 1 TURN

1B1

MAGNETIC FUNDAMENTALS



∫Edt=ø·N (Faraday's Law)

I=Hℓ/N (Ampere's Law)

SLOPE = L = ∫ Edt/I

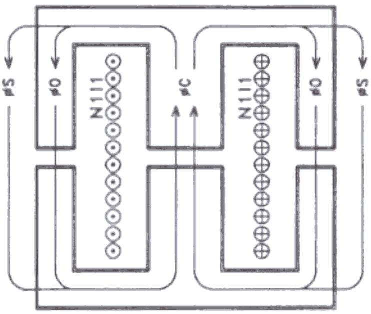
1B2

CONVERSION FACTORS, CGS to SI

		SI	CGS	CGS to SI
FLUX DENSITY	B	Tesla	Gauss	10^{-4}
FIELD INTENSITY	H	A–T/m	Oersted	$1000/4\pi$
PERMEABILITY (space)	μ_o	$4\pi\cdot10^{-7}$	1	$4\pi\cdot10^{-7}$
PERMEABILITY (Rel)				1
AREA (Core,Window)	A	m	cm	10^{-4}
LENGTH (Core, Gap)	l	m	cm	10^{-2}
TOTAL FLUX = ∫BdA	ø	Weber	Maxwell	10^{-8}
TOTAL FIELD = ∫Hdℓ	F, MMF	A–T	Gilbert	$10/4\pi$
RELUCTANCE = MMF/ø	R			$10^{9}/4\pi$
PERMEANCE=1/R=L/N^2	P			$4\pi\cdot10^{-9}$
INDUCTANCE = P·N^2	L	Henry	(Henry)	1
ENERGY	W	Joule	Erg	10^{-7}

1C

INDUCTOR — MAGNETIC STRUCTURE

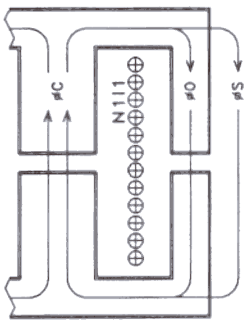


PROBLEMS:
DISTRIBUTED MMF
DISTRIBUTED FLUX
FRINGING FIELD
STRAY FLUX

SIMPLIFY:
COMBINE OUTER LEGS
  Symmetry
  No Fract. Turns

2

INDUCTOR — MAGNETIC STRUCTURE

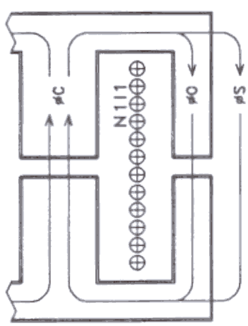


DISTRIBUTED MMF
CIRCULATION

SIMPLIFY:
CONCENTRATE WINDING
IGNORE FRINGING FIELDS
MIN # OF FLUX DIV. POINTS
DEFINE DISCRETE REGIONS

2A

INDUCTOR — MAGNETIC STRUCTURE



ASSUME TYPICAL VALUES:
$A_e = 1\,cm^2$; $\ell_e = 10\,cm$
$A_w = 2\,cm^2$ (ETD34)

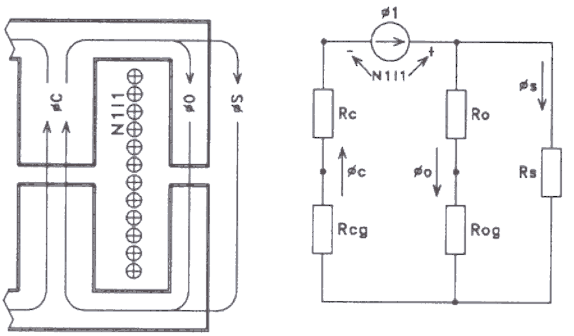
MAX $H\ell = NI_m = J_m A_w = 400 \cdot 2 = 800$

MAX ENERGY — FULL UTILIZ:
$W_m = H\ell \cdot B_m A_e / 2$
$= 800 \cdot .25 \cdot .0001/2 = 5\,mJ$

GAP LENGTH — FULL UTILIZ:
$\ell_g = NI/H = NI\mu/B$
$= 800 \cdot 4\pi \cdot 10^{-7}/.25 = .4\,cm$

2A1

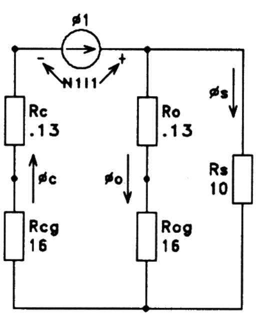
SIMPLIFIED RELUCTANCE MODEL



"DISCRETIZED" – SOURCE, Rc, Ro

2B

SIMPLIFIED RELUCTANCE MODEL



RELUCTANCE = MMF/ø

$R = H\ell/BA = \ell/\mu A$

APPORTION ℓe TO Rc, Ro

APPTN ℓg TO Rcg, Rog

$\mu = \mu o \mu r$, $\mu o = 4\pi \cdot 10^{-7}$

$\mu r = 1$ (air), $=3000$ (ferrite)

$Ro = (\ell e/2)/(\mu o \mu r Ae)$

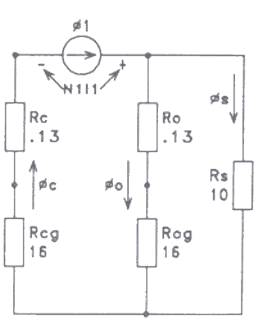
$= .05/(\mu o \mu r \cdot .0001)$

$Ro = .133 \cdot 10^{6}$ (omit 10^{6})

Rs IS AN ESTIMATE

2B1

SIMPLIFIED RELUCTANCE MODEL



WITH ALL LEGS GAPPED:

Rc<<Rcg; Ro<<Rog

STRAY FLUX, L>PREDICTED

WITH OUTERLEG GAP ONLY:

Rcg=0; Rc,Ro<<Rog,Rs

GREATER STRAY FLUX

WITH CENTERLEG GAP ONLY:

Rog=0

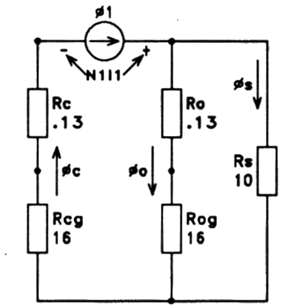
Ro<<Rcg and Rs

MINIMAL STRAY FLUX

Rcg TOTALLY DOMINANT

2B2

SIMPLIFIED RELUCTANCE MODEL



CALCULATING:
I1 FROM AMPERE'S LAW
V1 FROM FARADAY'S LAW
IS THIS THE EQUIVALENT ELECTRICAL CIRCUIT?

PROBLEM:
I1 IS A POTENTIAL
V1 IS A CURRENT

FOUR PROBLEMS

2B3

THE EQUIVALENT ELECTRICAL CIRCUIT

IS A DUAL OF THE MAGNETIC CIRCUIT

(ELECTRICAL DUALS ARE NOT EQUIVALENT: CUK vs FLYBACK)

MAGNETIC – ELECTRICAL DUALITY:

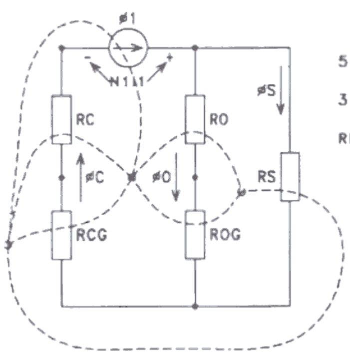
NODES	MESHES (LOOPS)
MMF	AMP–TURNS
dø/dt	VOLTS/TURN
RELUCTANCE	PERMEANCE
SHORT	OPEN
SERIES ADDN.	PAR. ADDN.

ORIENTATION: ROTATE IN SAME DIRECTION

CIRCUITS MUST BE PLANAR

2C

THE MAGNETIC/ELECTRICAL DUAL



5 NODES ---> 5 LOOPS

3 LOOPS ---> 3 NODES

RELUCTANCE-->PERMEANCE

MMF--->NI

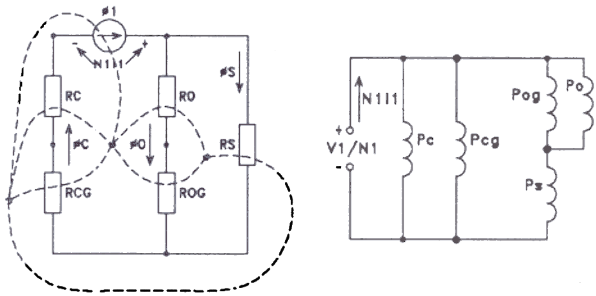
dø/dt-->V/N

ORIENTATION

PLANAR

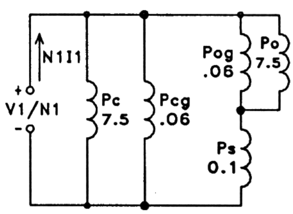
2C1

THE MAGNETIC/ELECTRICAL DUAL



2C2

THE EQUIVALENT ELECTRICAL CIRCUIT



PERMEANCE = 1/RELUCTANCE

UNITS: Henrys/Turn

$P = 1/R = \mu A/\ell = L/N^2$

$Po = 1/Ro = 7.5\mu H/N^2$

$V1/N1 = d\phi 1/dt$

$N1I1 = H\ell$

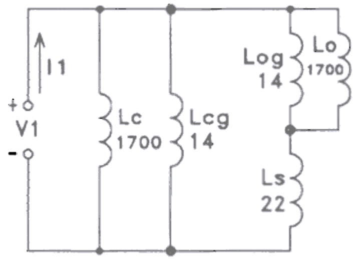
COMBINE INTO ONE

NOTE RELATIVE SIGNIFICANCE

2D

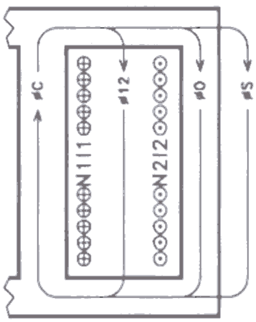
THE EQUIVALENT ELECTRICAL CIRCUIT

WITH N = 15 TURNS



2D2

SIMPLE TRANSFORMER



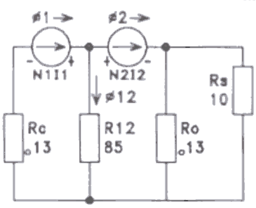
ASSUME V1 ACROSS N1
NO LOAD – I2=0
MMF IS TINY – NO GAP
I1=MAGNETIZING I_m
ø12 IS NEGLIGIBLE

WITH FULL LOAD I2
LARGE MMFs – CANCEL
EXCEPT BETWEEN COILS
ø12 IS LARGE
LEAKAGE L ENERGY
N1I1 = N2I2 + N1Im

3

TRANSFORMER RELUCTANCE MODEL

ETD34 CORE. Rc, Ro, Rs FROM PREVIOUS EXAMPLE
WINDOW DIM.: BREADTH bw=2.5cm, HEIGHT hw=.8cm
MEAN TURN LENGTH, MLT=6cm

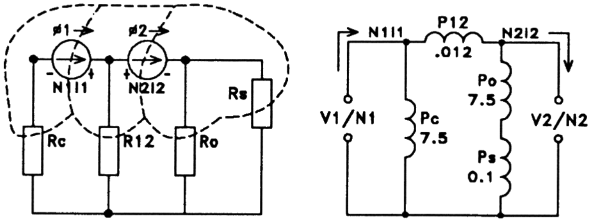


CALCULATE $R12 = \ell/\mu A$
ASSUME WDG CTR = .4cm
$A = .4 \cdot 6 = 2.4$ cm
$= .0024$ m
$\ell = 2.5\text{cm} = .025\text{m}$
$R12 = 85 \cdot 10^6$

ALL R VALUES $\cdot 10^6$

3A

THE TRANSFORMER ELECTRICAL DUAL

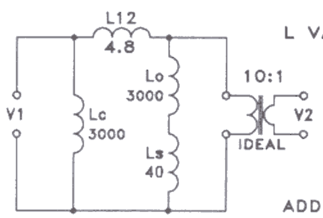


4 LOOPS, 4 NODES IN BOTH

3B

TRANSFORMER EQUIVALENT CIRCUIT

WITH Np = 20 TURNS, Ns = 2 TURNS



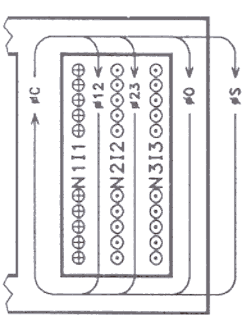
L VALUES REFERRED TO PRI.

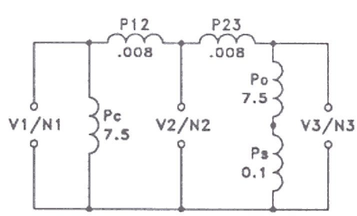
ADDED IDEAL TRANSFORMER
ALLOWS DIFFERENT Np/Ns
PROVIDES GALV. ISOLATION

3C

THREE–WINDING TRANSFORMER

Refer to Topic P2





PERMEANCE IN μH/turn^2

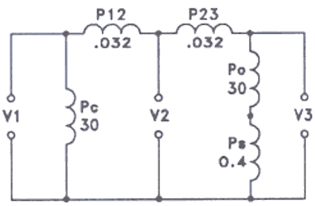
4

THREE–WINDING TRANSFORMER

ASSUME Np=20, Ns=2

L VALUES IN μH

REF TO 2 TURN SEC.



EXPLORE:

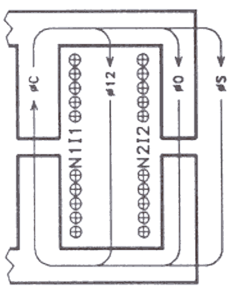
1 PRI (N1), 2 SEC (N2,N3)	T1=10:1
1 PRI (N1), 1 SEC (N2 OR N3)	T1=10:1
2 PAR PRI (N1,N2), 1 SEC (N3)	T1,T2=10:1 P
2 PAR PRI (N1,N3), 1 SEC (N2)	T1,T3=10:1 P
2 SER PRI (N1,N3), 1 SEC (N2)	T1,T3=5:1 S

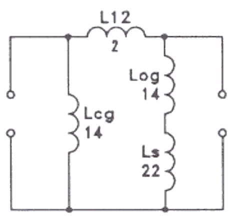
4A

COUPLED INDUCTOR

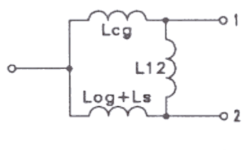
N1, N2 = 15 TURNS

L IN μH





Ref.
Topics
M7. P3



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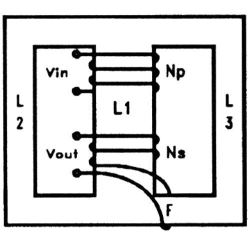
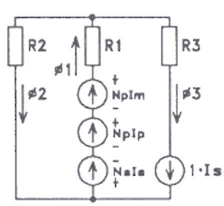
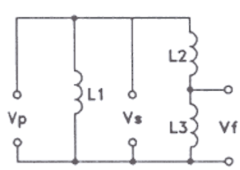
FRACTIONAL TURNS







EFFECTIVE LEAKAGE L

HOW TO STIFFEN UP

Ref. Topic M8

6

REFERENCES

1. CHERRY, E.C., "The Duality Between Electric and Magnetic Circuits and the Formation of Transformer Equivalent Circuits," Proc. Phys. Soc. (Britain), 62B, pp 101–111, Feb. 1949

2. Severns and Bloom, "Modern DC–to–DC Switchmode Power Converter Circuits," Van Nostrand Reinhold Co., Inc., New York, 1985.

3. Dauhajre and Middlebrook, "Modelling and Estimation of Leakage Phenomena in Magnetic Circuits," IEEE Power Electronics Specialists Conference, 1986 Record, pp. 213–226.

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