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Circuit Sensitivity With Emphasis On Analog Filters

Mark Fortunato Texas Instruments

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Circuit Sensitivity: Overview

- In audio applications, much of what is done in the analog domain is amplification and filtering
- Conflicting requirements for low cost and precision create difficult trade-offs
- A good understanding of circuit sensitivity provides a means to make these trade-offs easier
- We will investigate circuit sensitivity using filter circuits that are common building blocks used in audio designs
- The techniques are general in nature and are applicable to virtually all analog circuits

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Circuit Sensitivity: Overview

- All Circuits Have Performance That Varies As The Values Of The Components Change
 - Temperature
 - Time (Aging)
 - Phase Of The Moon
 - Component Tolerance In Manufacturing
 - Etc

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Circuit Sensitivity: Overview

- Op Amp Based Analog Filter Performance Is Largely Determined By The Passive Components
 - As Long As The Op Amp Is Adequate
 - Sufficient Bandwidth
 - Sufficient Gain In The Frequency Band Of Interest
 - Purchased From TI
 - Etc

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Circuit Sensitivity: Overview Of Filters

- Filter Design With Op Amps Involves Three Main Steps
 - Determining The Desired Characteristic
 - Determining The Mathematical Function For A Filter That Will Meet These Requirements
 - Have To Consider Performance Variation And Provide Enough Guard-band To Still Meet Requirements
 - Synthesizing A Circuit To Implement This Mathematical Function

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Circuit Sensitivity: Filter Synthesis

- Filter Circuit Synthesis: Designing A Circuit That Implements The Poles And Zeros Called For In The Mathematical Description
 - There Are Many Standard Topologies To Choose From
 - All With Well Document Equations
 - Sallen-Key
 - MFB
 - State-Variable
 - Biquad
 - Many, Many Others
 - Typically Split The Circuit Into Second Order Sections
 - Each Section Is Synthesized Independently
 - May Use The Same Topology For Each Section, Or Different

Actual Component Values Are Determined At This Stage
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Sensitivity: The Reality Sets In

- Real Filters Are Implemented With Real Components
 - Their Values Will Vary
 - From Unit To Unit
 - Over Environmental Conditions
 - Over Time
 - The Circuit Characteristics Are Dependent Upon These Values
 - Different Filter Topologies Have Different Sensitivities To Component Variations
 - Even Within A Single Topology, Sensitivities Can Vary As A Function Of The Component Values

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Sensitivity: The Reality Sets In

- How Much A Circuit's Behavior Changes As A
 Component Value Changes Is The Sensitivity
 - It Is Important To Know The Sensitivities For:
 - Proper Component Selection
 - i.e. A 1% Resistor vs. A 5% Resistor
 - Proper Choice Of Topology
 - May Need To Upgrade To A Less Sensitive Topology
 - Proper Guard-band In the Mathematical Description
 - If The Variations Are Too Great, May Have To Modify The Mathematical Description For Greater Margin

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Sensitivity: What Is It?

- Sensitivity: How Much One Thing Changes As A Function Of Another Thing That Is Changing
 - Changes In An Entire Transfer Function Relative To A Component Variation
 - Changes In An Individual Pole Or Zero Relative To A Component Variation
 - Changes In ω_n And Q Of Second Order Sections Relative To A Component Variation
 - This One Is Very Useful And Easy To Work With For Filter Design

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Sensitivity: What Is It?

Mathematical Definition Of Sensitivity:

$$S_{x}^{y} = \frac{\lim}{\Delta x \to 0} \left\{ \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} \right\} = \frac{x}{y} \frac{\partial y}{\partial x}$$

– Where X Is The Component That Is Varied And Y Is The Filter Characteristic (ω_n Or Q In Our Case) That We Wish To Evaluate As X Is Varied

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Sensitivity: Second Order Function

 A Second Order LPF Has This Generic Transfer Function

$$\frac{V_{out}}{V_{in}} = \frac{1}{S^2/\omega_n^2 + S/Q\omega_n + 1}$$

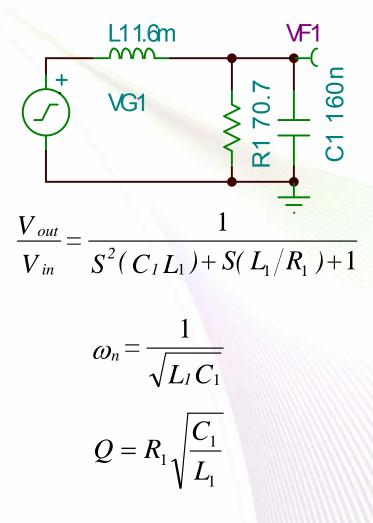
– Some people like to use the "damping factor", ζ , rather than Q

$$\zeta = \frac{1}{2Q}$$

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Sensitivity: A Simple Example

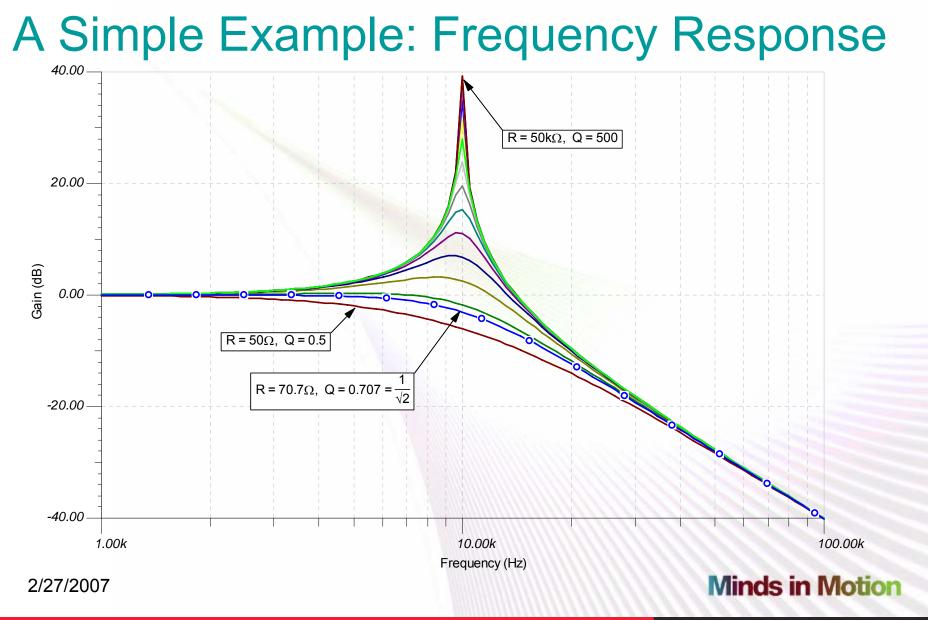


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A Simple Example: The Sensitivities

• What Are The Q And ω_n Sensitivities For This Circuit?

$$S_{CI}^{Q} = -S_{L1}^{Q} = \frac{1}{2}$$
$$S_{R1}^{Q} = 1$$
$$S_{CI}^{\omega_{n}} = S_{L1}^{\omega_{n}} = -\frac{1}{2}$$
$$S_{RI}^{\omega_{n}} = 0$$

• They Are The Exponents In The Circuit Equations:

$$\omega_n = \frac{1}{\sqrt{L_1 C_1}} \qquad \qquad Q = R_1 \sqrt{\frac{C_1}{L_1}}$$

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A Simple Example: The Sensitivities

- These Simple Results Help In Understanding What Sensitivities Mean
- EXAMPLE: If C1 Is Increase By A Factor Of 4

$$- \omega_n$$
 Changes By

– Q Changes By

$$4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}$$

 $4^{\frac{1}{2}} = 2$

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A Simple Example: Transfer Function Sensitivity

- We Can Also Calculate The Transfer Function Sensitivities
 - Using The Defining Equation For Sensitivity, The Sensitivity To C1 Is

$$S_{C1}^{H(s)} = \frac{C1}{H(s)} \frac{\partial H(s)}{\partial C1}$$

– After Proper Application Of Calculus, This Becomes

$$S_{C1}^{H(s)} = -\frac{s^2 L 1 C 1}{S^2 L 1 C 1 + S L 1 / R 1 + 1}$$

- Similarly, The Sensitivities To L1 and R1 Are

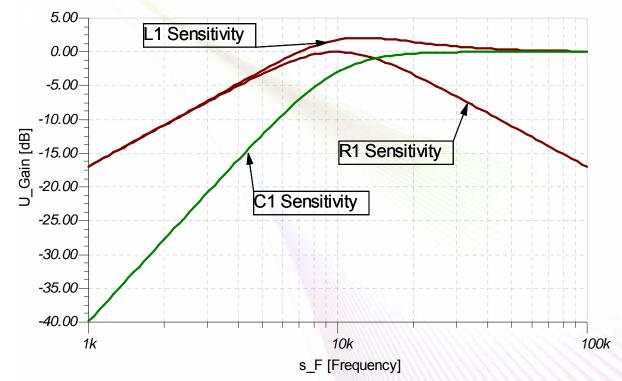
 $S_{L1}^{H(s)} = -\frac{s^2 L 1 C 1 + s L 1 / R 1}{S^2 L 1 C 1 + S L 1 / R 1 + 1} \qquad S_{R1}^{H(s)} = \frac{s L 1 / R 1}{S^2 L 1 C 1 + S L 1 / R 1 + 1}$

- These Results Are Far More Complex, And Thus Difficult To Put To Use
 - They Are Functions Of The Components AND of Frequency

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TI Developer Conference A Simple Example: Transfer Function Sensitivity

 We Can Plot The Sensitivity Functions Using The Nominal Component Values

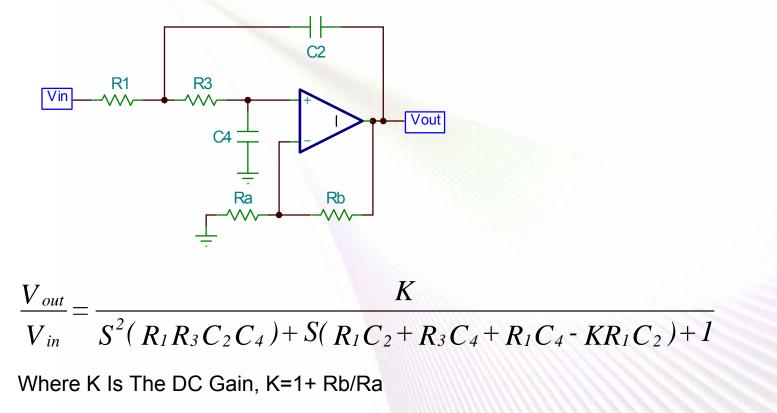


- There is a lot of good information in these plots, but we get all we need for most situations with the far simpler Q and ω_n sensitivities 2/27/2007 Minds in Motion

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Sensitivity: Active Filters

- The Equations Can Be More Complex For Active Filters
 - The Sallen-Key Filter Has Been Around For 50 Years (ref 1):



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Sensitivity: Active Filters

• The Characteristic Equations For This Filter Are

$$\omega_n = \frac{1}{\sqrt{R_1 R_3 C_2 C_4}} \qquad \frac{1}{Q} = \sqrt{\frac{R_3 C_4}{R_1 C_2}} + \sqrt{\frac{R_1 C_4}{R_3 C_2}} + (1 - K) \sqrt{\frac{R_1 C_2}{R_3 C_4}}$$

• The Sensitivities Of Q And ω_n Are

$$S_{RI}^{Q} = -S_{R3}^{Q} = -\frac{1}{2} + Q \sqrt{\frac{R_{3}C_{4}}{R_{1}C_{2}}}$$

$$S_{C2}^{Q} = -S_{C4}^{Q} = -\frac{1}{2} + Q \left(\sqrt{\frac{R_{1}C_{4}}{R_{3}C_{2}}} + \sqrt{\frac{R_{3}C_{4}}{R_{1}C_{2}}}\right)$$

$$S_{K}^{Q} = QK \sqrt{\frac{R_{1}C_{2}}{R_{3}C_{4}}}$$

$$S_{RA}^{Q} = -S_{RB}^{Q} = -Q(K-1) \sqrt{\frac{R_{1}C_{2}}{R_{3}C_{4}}} = -Q \frac{R_{B}}{R_{A}} \sqrt{\frac{R_{1}C_{2}}{R_{3}C_{4}}}$$

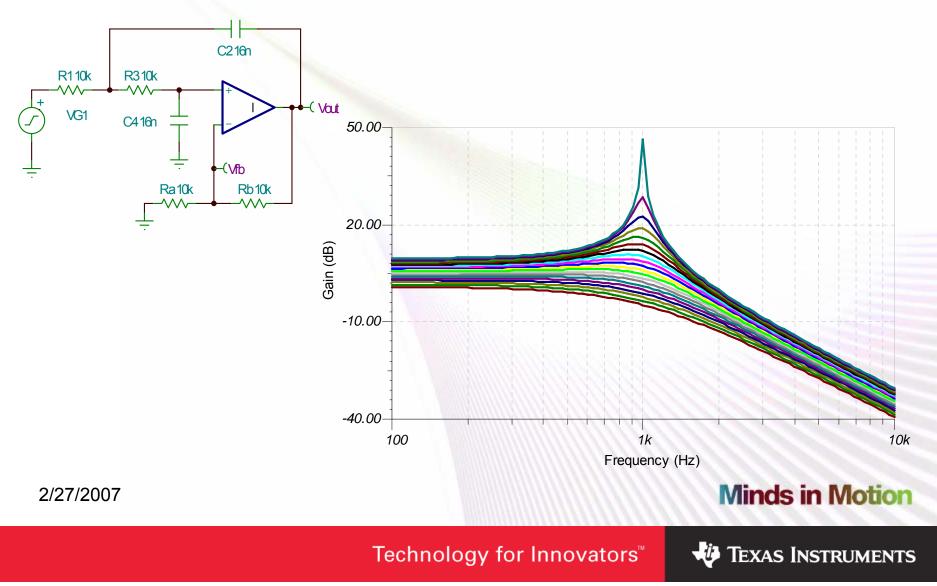
$$S_{R1}^{Q_{n}} = S_{R3}^{Q_{n}} = S_{C2}^{Q_{n}} = S_{C4}^{Q_{n}} = -\frac{1}{2}$$

$$S_{K}^{Q_{n}} = S_{Ra}^{Q_{n}} = S_{Rb}^{Q_{n}} = 0.$$

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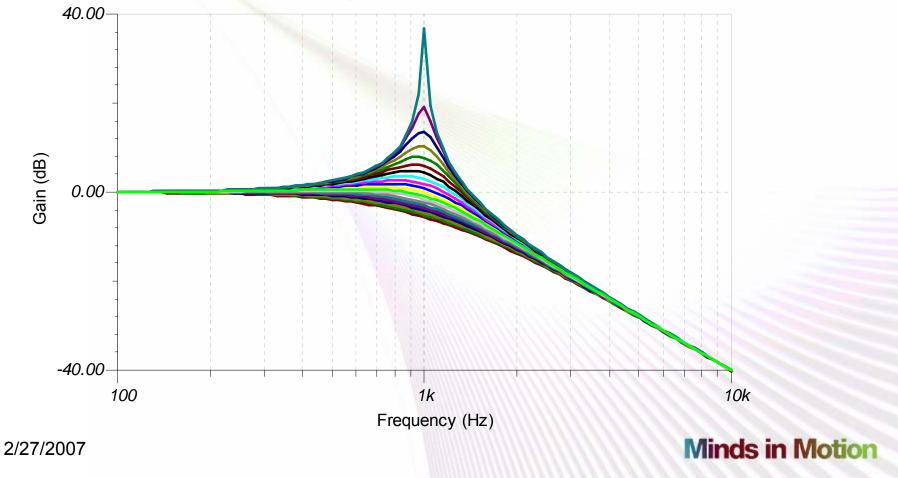
Active Filter: Simple Example

• 1kHz, Second Order LPF: Vary Rb From 1k To 19.9k



Active Filter: Simple Example

 Monitor The Voltage At The Inverting Terminal For Gain-Normalized Response



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Sensitivity: Are We Done?

- We Can Determine How Important Each Component Is To Each Circuit Parameter With The Sensitivity Equations
 - This Allows Us To Intelligently Specify The Precision Of The Components We Selected

Is This All We Can Use Sensitivity Analysis For?

- We Can Also Use The Sensitivity Equations To Select Component Values To Reduce Sensitivity
 - Sensitivity Equations That Are Functions Of Component Values Often Reveal Trade-Offs That We Can Make

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Sensitivity: Goin' Back For More

- Reconsider The Equations For Our Active Filter
 - Notice That At K=1 The Q sensitivity To Ra and Rb Goes to Zero
 - With K=1 The Equation For Q Simplifies

$$\frac{1}{Q} = \sqrt{\frac{R_3 C_4}{R_1 C_2}} + \sqrt{\frac{R_1 C_4}{R_3 C_2}} + (1 - K)\sqrt{\frac{R_1 C_2}{R_3 C_4}} \longrightarrow \frac{1}{Q} = \sqrt{\frac{R_3 C_4}{R_1 C_2}} + \sqrt{\frac{R_1 C_4}{R_3 C_2}}$$

- And The Sensitivity Equations Simplify To

$$S_{RI}^{Q} = -S_{R3}^{Q} = -\frac{1}{2} + \frac{R_{3}}{(R_{3} + R_{1})}$$
$$S_{C2}^{Q} = -S_{C4}^{Q} = \frac{1}{2}$$

$$S_{K}^{Q} = Q \sqrt{\frac{R_{1}C_{2}}{R_{3}C_{4}}}$$

$$S_{RA}^{Q} = -S_{RB}^{Q} = 0$$

$$S_{R1}^{\omega_{n}} = S_{R3}^{\omega_{n}} = S_{C2}^{\omega_{n}} = S_{C4}^{\omega_{n}} = -\frac{1}{2}$$

$$S_{K}^{\omega_{n}} = S_{Ra}^{\omega_{n}} = S_{Rb}^{\omega_{n}} = 0.$$

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Sensitivity: Goin' Back For More again

- While Q Sensitivity to Ra & Rb is 0, Q sensitivity to K Is Not
 - In Fact the Sensitivity of Q to K is Quite High for High Q Stages
 - The Sensitivity is Proportional to Q & Proportional to the Square-Root of the Larger Capacitor Divided by the Smaller Capacitor
 - These Two Factors Combined Make The Sensitivity Proportional to Q²
 - With A Perfect OpAmp, K Will Always Be Precisely 1
 - So It Doesn't Matter What The Sensitivity Is
 - But A Real OpAmp Has Finite Open-Loop Gain
 - At Frequencies With Sufficient Loop Gain, Reduction of K Will Be Negligible
 - At Higher Frequencies the Loop Gain Will Be Low Enough That Reduction in K Will Noticeably Affect Q
 - We Will Look at the Practical Implications of This Factor Shortly



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Sensitivity: Another Reduction

- A Further Simplification Occurs If We set R1=R3
 - This Results In The Q Sensitivity to R1 and R3 to Reduce to:

$$S_{RI}^{Q} = -S_{R3}^{Q} = -\frac{1}{2} + \frac{R_{3}}{(R_{3} + R_{1})} = -\frac{1}{2} + \frac{1}{2} = 0$$

- These Sensitivities Are Only Zero When R3=R1 Exactly
 - Sensitivity With 5% Resistors

$$S_{RI}^{Q} = -S_{R3}^{Q} = -\frac{1}{2} + \frac{R_3}{(R_3 + R_1)} = -\frac{1}{2} + \frac{1.05}{(1.05 + 0.95)} = 0.025$$

- This Is Quite Insensitive. We May As Well Think Of It As 0%

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Sensitivity Reduction: What Have We Done?

• With This Simplification The Sensitivity Equations Are Now:

$$S_{RI}^{Q} = S_{R3}^{Q} = 0$$

$$S_{C2}^{Q} = -S_{C4}^{Q} = \frac{1}{2}$$

$$S_{K}^{Q} = Q \sqrt{\frac{C_{2}}{C_{4}}}$$

$$S_{RI}^{\omega_{n}} = S_{R3}^{\omega_{n}} = S_{C2}^{\omega_{n}} = S_{C4}^{\omega_{n}} = -\frac{1}{2}$$

$$S_{K}^{\omega_{n}} = 0$$

- We Have Reduced All Sensitivities to ½ Or Zero Except For Q Sensitivity to K; The Parameter That Varies The Least
 - This Is Similar To Noise Shaping In a Sigma-Delta Converter Where We Move Most Of The Noise Out Of The Band Of Interest
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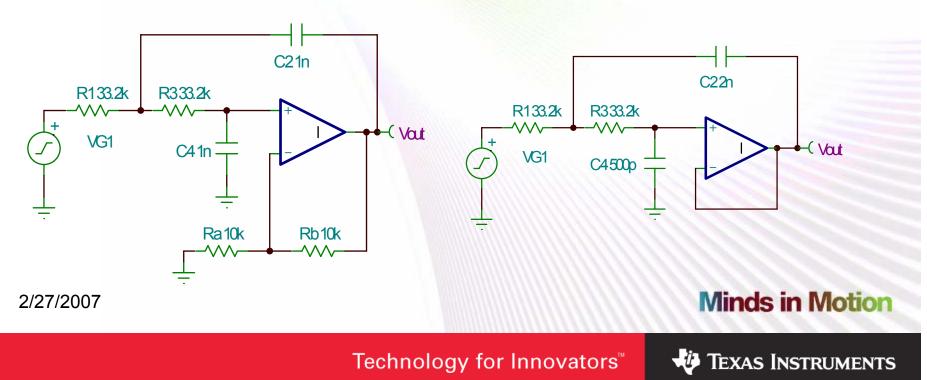
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Sensitivity: Let's Get Practical

• Implement A 2nd Order LPF With:

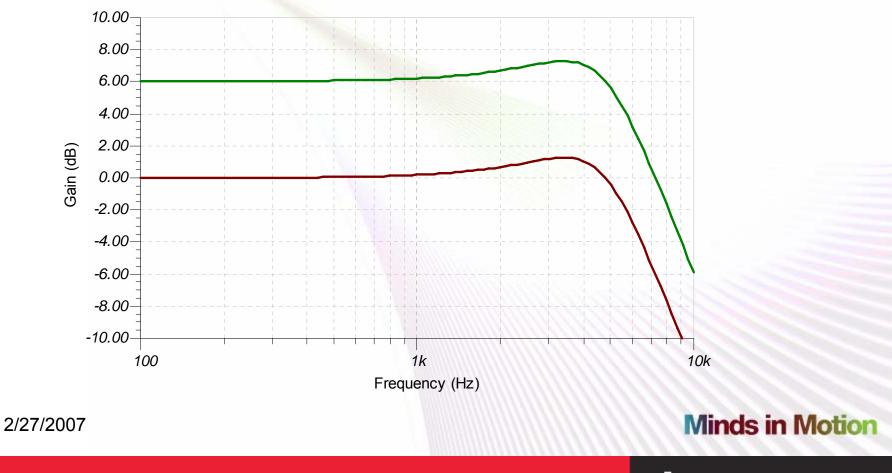
 $f_n = 4.8 kHz$ Q = 1

- Use A "Cookbook" Approach From Reference 2, pp156-7
 - Choose 1nF For The Capacitors And Crunch Through The Procedure
- Now Implement This Filter With The Low Sensitivity Version
 - Start With The Same Value Resistor, 33.2k



Let's Get Practical: Frequency Response

• The Nominal Responses Are The Same With A 6-dB Offset

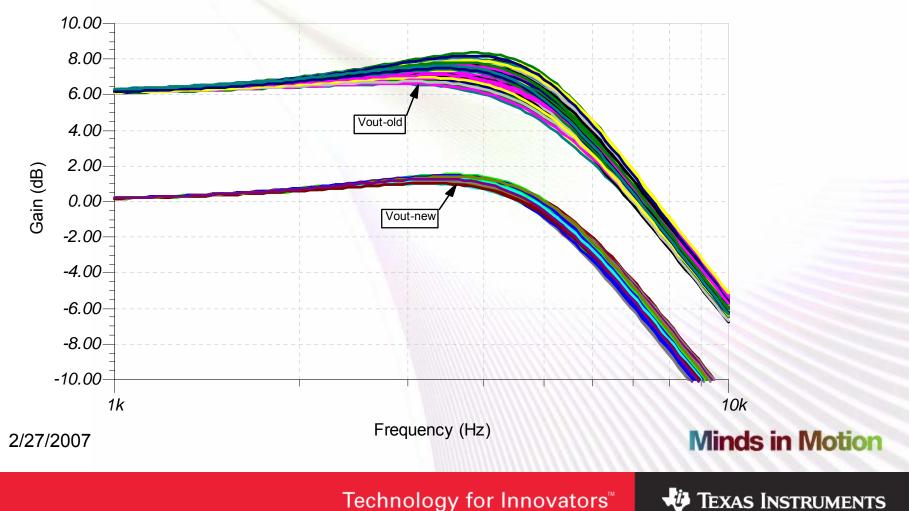


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Let's Get Practical: Frequency Response

• Use 5% Capacitors And 1% Resistors And Let The Components Vary: Monte Carlo Analysis With 100 Runs



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Low Sensitivity S-K: What Did We Give Up

- What Are The Trade-Offs We Make For This Improvement In Sensitivity?
 - We Can Only Get A Gain Of 1 For Any Stage
 - The Capacitor Values Can Get Unmanageable For High Qs
- The Equation For Q Has Been Simplified To:

$$\frac{1}{Q} = 2\sqrt{\frac{C_4}{C_2}} \qquad \text{OR} \qquad Q = \frac{1}{2}\sqrt{\frac{C_2}{C_4}}$$

– The Capacitor Values Are Related By This Equation:

$$C_2 = 4Q^2 C_4$$
 OR $\frac{C_2}{C_4} = 4Q^2$

- For A Q Of 10, The Capacitor Value Ratio is 400

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Low Sensitivity S-K: What Did We Give Up

- If We Do Not Restrict The Resistors To Being Of Equal Value Can We Reduce This Capacitor Ratio?
 - Q is Defined By The Following Equation

$$Q = \frac{1}{\sqrt{\frac{R_{3}C_{4}}{R_{1}C_{2}}} + \sqrt{\frac{R_{1}C_{4}}{R_{3}C_{2}}}}$$

– Let's Call R3/R1 = n and C4/C2 = m. Then This Equation Becomes:

$$Q = \frac{1}{\sqrt{nm} + \sqrt{\frac{m}{n}}}$$

 If R3=R1 Gives the Highest Q Then The Derivative of This Equation Must be Zero When n=1

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Low Sensitivity S-K: What Did We Give Up

- The Derivative of The Denominator Would Also Be Zero as the Denominator Would be at a Minimum When R1=R3
- The Denominator is

$$\sqrt{nm} + \sqrt{\frac{m}{n}}$$

• It's Derivative is

$$\frac{1}{2}\sqrt{\frac{m}{n} - \frac{1}{2}\frac{\sqrt{m}}{\frac{3}{n^2}}} = \frac{1}{2}\sqrt{m} - \frac{1}{2}\sqrt{m} = 0 \quad When \quad n = 1$$

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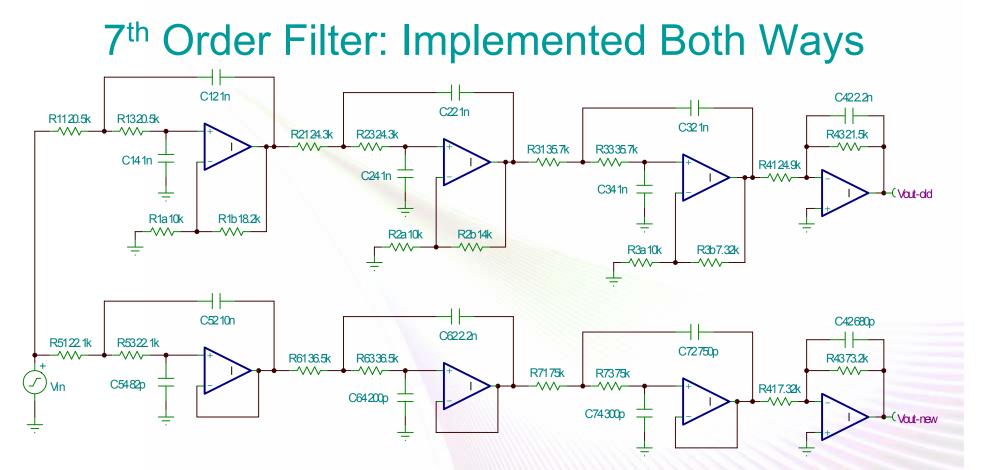
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Low Sensitivity S-K: How Far Can We Go?

- Limiting The Maximum Q To About 5, We Can Still Implement Fairly Complex Filters
 - Let's Implement A filter Meeting These Requirements
 - 7th-Order Chebychev
 - 0.05dB Nominal In-Band Ripple
 - 8kHz 3-dB Frequency
 - Gain = 10
 - The Poles Required For This Filter Are:

Fn	Q
7.834kHz	5.5662
6.560kHz	1.6636
4.492kHz	0.7882
3.162kHz	simple

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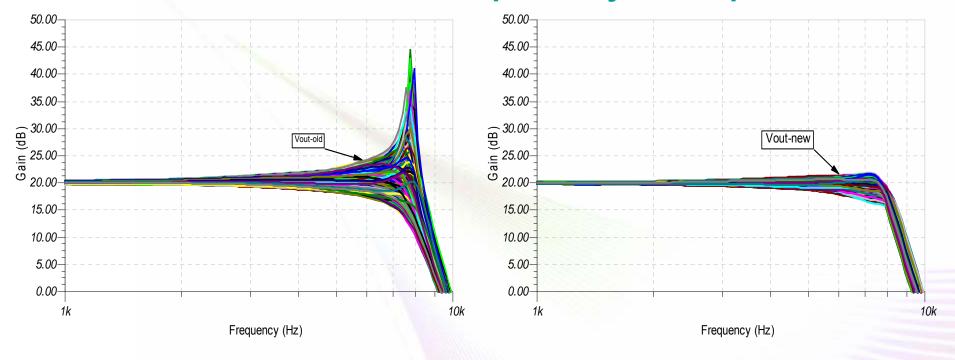
The Cookbook Version (Top) Has Capacitor Values From 1nF to 2.2nF While The Low-Sensitivity Sallen-Key Version Capacitors Vary From 82pF to 10nF

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7th Order Filter: Frequency Responses



This Cookbook Version Has Over 30dB Variation Near f_n

This Low-Sensitivity Version Has About 5dB Variation Near f_n

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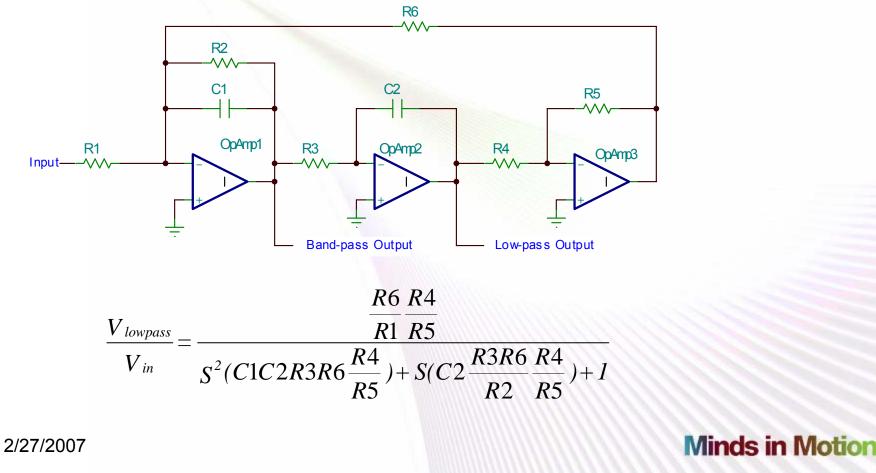
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7th Order Filter: The Lazy Way

- Some Filter Topologies Have Inherently Low Sensitivities
 - The 3-Op Amp Biquad Is A Common Example



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7th Order Filter: The Lazy Way Q, ω_n & Sensitivities Can Be Calculated As: $\omega_n = \frac{1}{\sqrt{C1C2R3R6\frac{R4}{R5}}} \qquad Q = \sqrt{\frac{C1R2^2}{C2R3R6}\frac{R5}{R4}}$ $S_{C1}^{\omega_n} = S_{C2}^{\omega_n} = S_{R3}^{\omega_n} = S_{R6}^{\omega_n} = S_{R4}^{\omega_n} = -S_{R5}^{\omega_n} = -1/2$ $S_{R2}^{Q} = 1$ $S_{C2}^{Q} = S_{R3}^{Q} = S_{R6}^{Q} = S_{R4}^{Q} = -S_{R5}^{Q} = -1/2$

All Sensitivities Are Independent Of Component Values

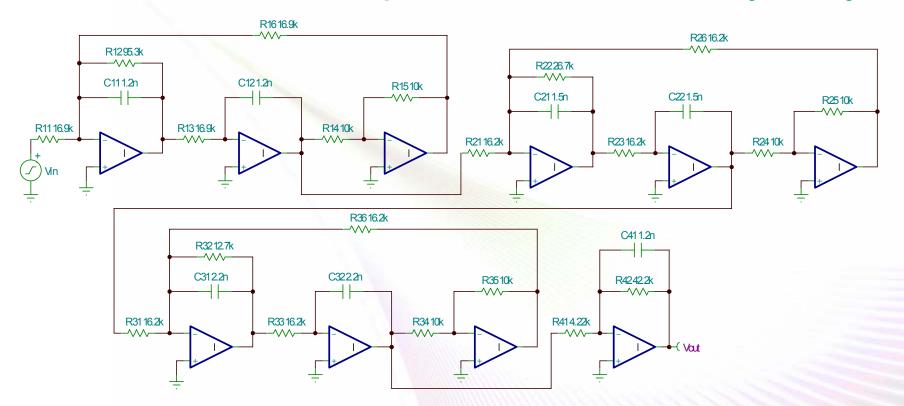
- You Can Chose Component Values Without Any Consideration Of How They Affect The Sensitivity Of The Circuit
- Can Generate Circuits With Higher Qs

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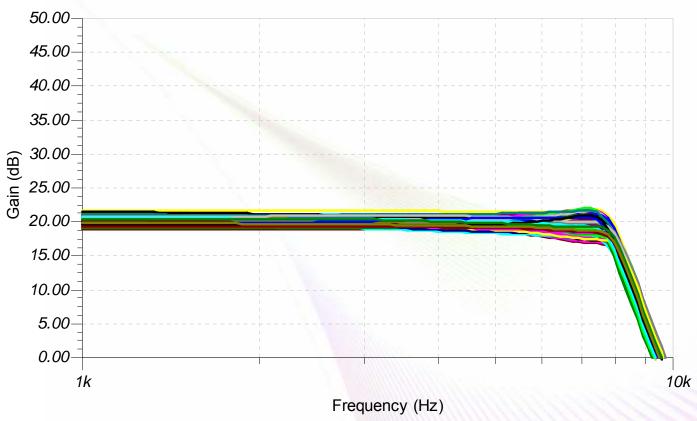
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7th Order Filter: Implemented The Lazy Way



- The Biquad Has 10 Op Amps, 20 Resistors & 7 Capacitors, Compared To 4 Op Amps, 8 Resistors & 7 Capacitors For The Low-sensitivity Sallen-Key Filter
- The Biquad Allows All Capacitor Values To Be Within A Narrow Range
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7th Order Filter: Lazy Way Frequency Response



- The Results Are Very Close To That Of The Low-Sensitivity Sallen-Key Circuit
 - However, The DC Gain Varies A Couple dB More In This Implementation

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DC Gain Variations: Sallen-Key

• For A Sallen-Key Filter Stage The DC Gain Is

$$H_0 = K = 1 + \frac{R_B}{R_A}$$

– The DC Gain Sensitivities Are Thus

$$S_{R_B}^{H_0} = -S_{R_A}^{H_0} = \frac{1}{1 + \frac{R_A}{R_B}}$$

All Other Sensitivities Are Zero

- For The Unity Gain Version
 - Ra Is Infinite
 - Rb is Zero
 - So All Gain Sensitivities Are Zero

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DC Gain Variations: Biquad

• For A Biquad Stage The DC Gain Is

$$H_{0} = \frac{R_{6}}{R_{1}} \frac{R_{4}}{R_{5}}$$

– The DC Gain Sensitivities Are Thus

 $S_{R6}^{H_0} = -S_{R_1}^{H_0} = S_{R_4}^{H_0} = -S_{R_5}^{H_0} = 1$

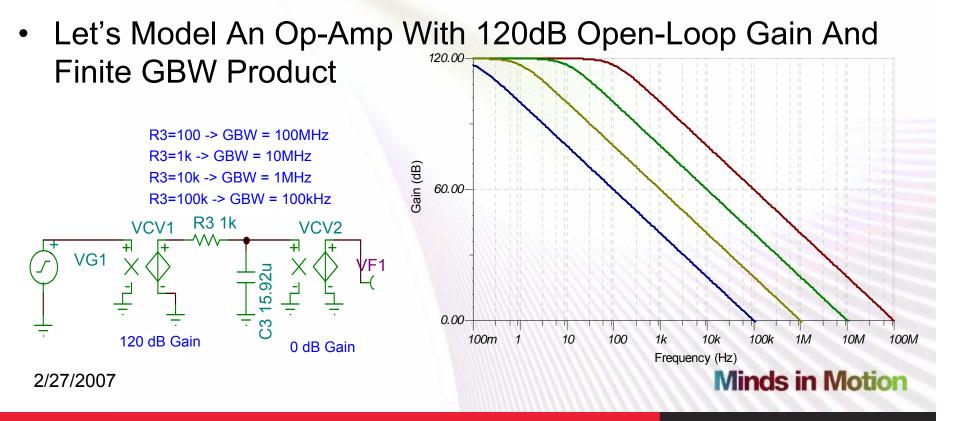
- All Other Sensitivities Are Zero
- There Is No Way To Eliminate The Gain Dependency On These Four Resistors, Thus There Will Always Be DC Gain Variations With This Topology

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What Happens With A Non-Ideal Op-Amp

- Recall That We Have to Be Careful To Have Adequate GBW to Not Run Into Problems With Q Sensitivity to K
 - A follower's Gain Decreases From 1 With Inadequate OpAmp GBW

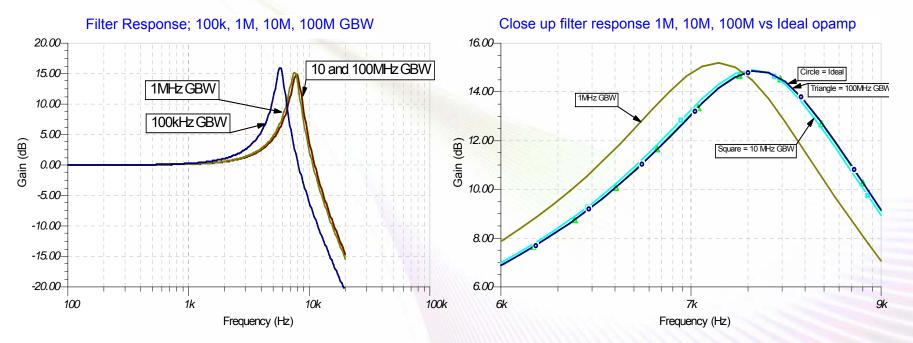


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Non-Ideal Op-Amp Filter Response

 Implementing The High-Q Stage of The 7th Order Filter With The Non-Ideal Op Amp, We See The Following



 Both 10M and 100M GBW Seem To Be Almost Identical To The Ideal OpAmp

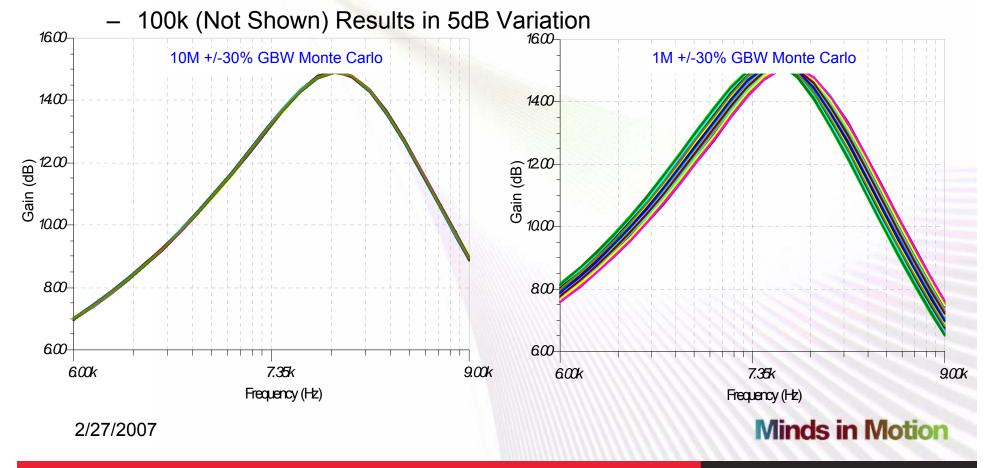
Monte Carlo Analysis Varying GBW Will Tell Us How Good These Are
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Non-Ideal Op-Amp Filter Response

- Vary GBW +/-30%, Typical For Real Op Amps
 - 10M GBW Results In Much Less Than 1dB Variation
 - 1M GBW Results In About 1 dB Variation



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Mixing Filter Topologies

- To Build Higher Order Filters With Qs Greater Than 5
 - Implement Stages With Qs Lower Than 5 With The Low-Sensitivity Sallen-Key
 - Implement Higher Q Stages With The Biquad Or Similar Topology
- Rarely Do Filters Have More Than One Stage With A High Q

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Sensitivity: What Have We Learned

- Calculating Q and ω_{n} Sensitivities Is Fairly Easy
- Being Aware Of Component Sensitivity We Can At Least Chose To Tighten Up On Tolerances For Components To Which The Circuit Response Is Most Sensitive
- By Using The Sensitivity Information Intelligently We Can Often Configure Circuits To Be Inherently Less Sensitive
- Understanding The Limitations Of These Less Sensitive Circuits, We Can Chose To Use More Complex Circuit Topologies Only When Necessary While Using Simpler, But More Limited In Usability, Insensitive Circuits For The Majority Of Needs
 - Trading off a little more design time for cost savings in components, test time, etc
- You Can Do A Lot With Tina, Including Generate Every Graphic In This Presentation

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Sensitivity: References

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